

# Centroidal Momentum Observer: Towards Whole-Body Robust Control of Legged Robots Subject to Uncertainties

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**Abstract**—This paper aims to provide robust dynamic motion control and eliminate parameter uncertainty in single-legged robots with a centroidal momentum-based control algorithm. In order to ensure the continuity of the movement despite external disturbances, the disturbance observer (DOB), which is frequently used in the motion control literature, is used. A one-legged robot model was simulated using MSC ADAMS, and then a centroidal momentum-based control algorithm and Zero Moment Point (ZMP)-based control algorithm were developed. The performances of the controllers were tested in the simulation environment in line with three different scenarios, unknown load, external forces and external momentum disturbances. The controllers were evaluated by comparing the selected reference orbital positions with the center of mass (CoM) positions on the x and z axes and the ZMP positions on the x-axis. As a result, it was possible to satisfactorily validate the proposed method, the centroidal momentum observer, and test the algorithm in the simulation environment. According to the simulation results, it was seen that the designed centroidal momentum observer worked in all tested conditions.

**Index Terms**—centroidal momentum control, legged robot

## I. INTRODUCTION

Humanoid robots and bipedal exoskeletons are used for human support. The behaviour and control of a one-legged robot can be used as a basis for each leg of multi-legged systems. These robots are used in a wide range from aircraft construction to disaster scenarios, from robotic rehabilitation to active walking support. [1]–[4].

Despite several studies, robustness of legged systems is still an active research question. Among various control strategies, linear inverted pendulum-model (LIPM) and Zero Moment Point (ZMP) based approaches have been successful in demonstrating versatile bipedal locomotion [5], [6]. For walking robots moving on flat horizontal surfaces, dynamic balance can be achieved using the principle of ZMP [6]. To this end, Kajita et al. have introduced a walking pattern generator that allows arbitrary foot placement in which they adapted the preview control theory [7]. Following a similar approach, Englsberger

et al. utilized divergent component of motion (also called capture point) to ensure dynamically balanced 3-D walking motion [8].

In pendulum-based or ZMP-based methods [5]–[7], it is assumed that an abstracted model represents the robot dynamics which may additionally require dynamics filtering [9]. Thus, optimization-based control methods emerged in recent years. In these methods, the controller is synthesized via solving a real-time optimal control problem in which an abstracted model is used to derive an approximate value function [10], [11]. Despite this fact, the method can still deal with the dynamic and contact constraints that arise due to full robot dynamics.

When regulating whole-body robot behavior, angular momentum appears to be an important quantity [12], [13]. Kajita et al. proposed resolved momentum control to automatically coordinate upper body and lower body movements [14]. Orin et al. derived the notion of centroidal momentum for a multi-body floating-base systems [15], [16] which led to the synthesis of synergistic whole-body humanoid control. [17]. Furthermore, recently developed optimization-based algorithms allow the centroidal momentum-based trajectory control in real-time [18], [19].

Note that, the notion of centroidal momentum is sensitive to parametric uncertainties, as it is a model-based algorithm. This creates a certain disadvantage in yielding robust locomotion control behavior, potentially limiting its applicability to settings with internal and external disturbances. Disturbance observer (DOB) is frequently applied and has been found to be effective in parameter uncertainties for motion control. It is a very useful robust control tool, owing to its efficacy and real-time applicability [20]–[22].

In light of these facts, we propose a method named Centroidal Momentum Observer (CMD), in which a disturbance observer block is adapted to a centroidal momentum-based locomotion control scheme. In this study, we aim to increase the robustness of against the external and internal disturbances when addressing balance control of legged systems. Therefore, the main contribution of this study is to pave the way for robust

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whole body control schemes for legged systems in which centroidal momentum control is synergistically combined with a disturbance observer scheme.

The remainder of this paper is organized as follows. In section II, centroidal momentum control and centroidal momentum observer are explained. In section III, simulation results are presented and discussed. Finally, conclusion are presented in section IV.

## II. CENTROIDAL MOMENTUM-BASED CONTROL

### A. Centroidal Momentum Matrix

The spatial momentum vector of a robot consists of linear and angular momentum. Following the derivation provided in [17], the spatial momentum concerning the  $i^{th}$  link is calculated as below:

$$h_i = \begin{bmatrix} k_i \\ l_i \end{bmatrix} = I_i v_i = I_i \begin{bmatrix} w_i \\ v_i \end{bmatrix} \quad (1)$$

In (1),  $h_i$  is the spatial momentum of the  $i^{th}$  link which consists of angular momentum  $k_i$  and linear momentum  $l_i$ . Likewise,  $v_i$  stands for the spatial velocity concerning the  $i^{th}$  link, including angular velocity  $w_i$  and translational velocity  $v_i$ . The spatial momentum can be obtained by multiplying  $v_i$  with the spatial inertia  $I_i$  which can be computed as below.

$$I_i = \begin{bmatrix} \bar{I}_i & m_i S(c_i) \\ m_i S(c_i)^\top & m_i \end{bmatrix} \quad (2)$$

As observed in (2),  $I_i$  is calculated using mass  $m_i$ , relative center of mass position  $c_i$ , and rotational inertia of the  $i^{th}$  link. Furthermore,  $S(c_i)$  is the skew symmetric matrix concerning the  $c_i$  vector. The centroidal momentum of a multi-dof system can be obtained through the sum of individual momenta of each link, provided that they are projected onto the CoM. To do so, we place a coordinate frame onto the CoM, named  $\{G\}$  which is oriented in an identical manner to fixed frame of reference (FFR)  $\{0\}$  [17]. Keeping this in mind, the sum of individual momenta can be computed as follows:

$$h_G = \sum_{i=1}^{N_b} {}^i X_G^\top h_i \quad (3)$$

In (3),  ${}^i X_G^\top$  is the spatial transformation matrix defined between the  $i^{th}$  link and  $\{G\}$ . It is computed as in the following:

$${}^i X_G^\top = \begin{bmatrix} {}^G R_i & {}^G R_i S({}^i p_G)^\top \\ 0 & {}^G R_i \end{bmatrix} \quad (4)$$

where  ${}^i R_G$  and  ${}^i p_G$  are the rotation matrix and position vector defined between the  $i^{th}$  link and  $\{G\}$ , respectively.

The centroidal momentum can also be expressed as follows,

$$h_G = A_G \dot{q}, \quad (5)$$

where  $A_G(q)$  is the centroidal momentum matrix, mapping  $h_G$  to  $\dot{q}$ . The vector  $\dot{q} \in \mathbb{R}^{(n+6)}$  stands for the generalized velocities, including actuated joints and floating coordinates.

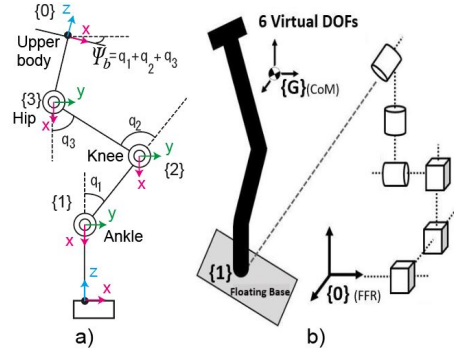


Fig. 1. The joint configurations of one-legged robot [25], and coordinate systems used.  $\{0\}$  is the fixed frame of reference (FFR),  $\{1\}$  is the floating base coordinate system that is fixed under the foot and, the center of gravity  $\{G\}$  coordinate system is attached to the CoM. This figure is adapted from [17].

One can also compute the net wrench acting on the CoM by differentiating (5):

$$f_G = \dot{h}_G = A_G \ddot{q} + \dot{A}_G \dot{q} \quad (6)$$

The main function of the CMM ( $A_G(q)$ ) is to provide a linear mapping between the generalized velocities and the centroidal momentum. To derive the ( $A_G(q)$ ) matrix, floating base dynamics can be used [23]. To this end, 6D vector approach is deemed to be computationally efficient [24]. For a general robotic system, the equation that governs rigid body dynamics is expressed as below,

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \quad (7)$$

where  $q \in \mathbb{R}^{(n+6)}$  stands for the generalized coordinates and  $M(q) \in \mathbb{R}^{(n+6) \times (n+6)}$  is the joint-space inertia matrix,  $C(q, \dot{q}) \in \mathbb{R}^{(n+6)}$  is the vector of Coriolis and the centrifugal terms,  $g(q) \in \mathbb{R}^{(n+6)}$  is the gravity term,  $\tau \in \mathbb{R}^{(n+6)}$  is the generalized force. The eq. (7), can be re-written to separately express floating-base and actual joint components [24]:

$$\begin{bmatrix} M_{11} & M_{1*} \\ M_{*1} & M_{**} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_* \end{bmatrix} + \begin{bmatrix} C_1 \dot{q} + g_1 \\ C_* \dot{q} + g_* \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_* \end{bmatrix} \quad (8)$$

In (8), the terms with the underscript  $\{1\}$  are associated with the floating base coordinates. Likewise,  $\tau_1$  is associated with floating base, such that  $\tau_1 = 0$  as they cannot produce force. Although the floating base is not actuated, the following equation is derived via the principle of virtual work [26]:

$$\Phi_1^\top f_1 = \tau_1 \quad (9)$$

In (9),  $\Phi_1$  can be derived via floating-base kinematics [17], [27], and it maps  $\tau_1$  to the net wrench acting on the floating base, namely  $f_1$ . Considering the gravitational force as an external force, the net wrench acting on the floating base can be yielded as in the following:

$$f_1 = (\Phi_1^\top)^{-1} U_1 (M(q)\ddot{q} + C(q, \dot{q})\dot{q}) \quad (10)$$

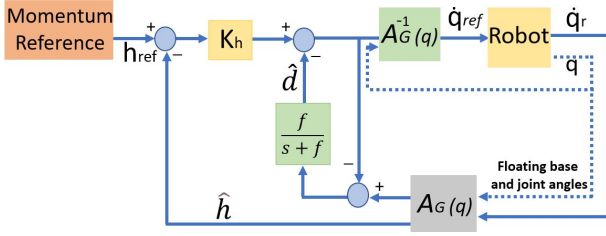


Fig. 2. Controller scheme of the proposed Centroidal Momentum Observer. The parameters with hat are obtained through estimation. The parameter  $f$  is low pass filter gain which is tuned empirically. The momentum reference is determined by trajectory planning.

In (10),  $U_1$  is defined as  $U_1 = [1_{6 \times 6} \ 0_{6 \times n}]$ . Using the spatial transformation matrix between the frames  $\{1\}$  and  $\{G\}$ , the net external wrench acting on the CoM can be obtained.

$$f_G = {}^i X_G^T f_1 \quad (11)$$

Combining (10) with (11), the following equation is yielded.

$$f_G = {}^i X_G^T (\Phi_1^T)^{-1} U_1 (M\ddot{q} + C\dot{q}) \quad (12)$$

$$= A_G \ddot{q} + \dot{A}_G \dot{q} \quad (13)$$

It is known that the eq. (13) holds true for an arbitrary choice of generalized acceleration,  $\ddot{q}$ . Therefore, the CMM  $A_G$  can be obtained as below.

$$A_G = {}^i X_G^T (\Phi_1^T)^{-1} U_1 M(q) \quad (14)$$

This result imply that one can compute the centroidal momentum using the generalized coordinates and velocities, which consist of floating base and actuated joint coordinates.

### B. Centroidal Momentum Observer

One possible drawback concerning the centroidal momentum-based control is model-dependency. Hence, it may not be able to deal with factors such as parameter uncertainty and internal disturbances. To remedy this issue, disturbance observer may be used as it is known to provide robustness even within the presence of modelling uncertainties [20], [28].

A simplified block diagram can be viewed in Fig. 2, where centroidal momentum feedback control and an associated disturbance observer is constructed. In centroidal momentum feedback control, one can obtain the reference joint velocities based on desired linear and angular momenta. For this operation, we need to use the inverse of  $A_G(q)$  matrix. Likewise, the inverse model includes the  $A_G(q)$  matrix itself. Furthermore, floating base coordinates must be measured and fed to the CMM matrix at all times. In practice, this could be achieved through the use of IMU sensors and state estimators [29].

In order to investigate the stabilization capability of the proposed centroidal momentum observer method, we use the one-legged robot model in Fig. 6. For this purpose, external angular momenta of -0.1 Nms, -0.3 Nms, -0.5 Nms, 0.1 Nms,

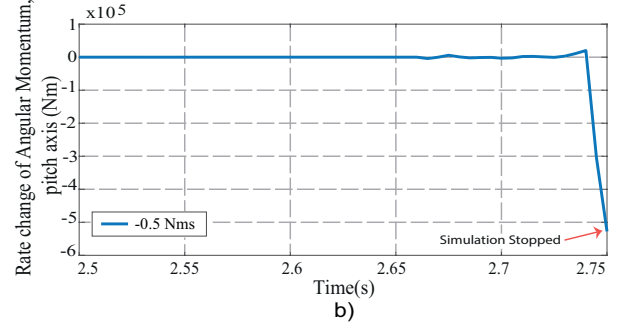
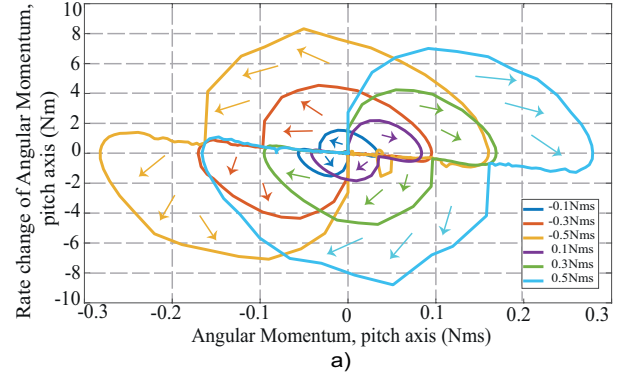


Fig. 3. Angular momentum phase portraits (a) with DOB. (b) without DOB.

0.3 Nms and 0.5 Nms are applied to the robot CoM when DOB is in effect in accordance with the controller displayed in Fig. 2. Moreover, we repeat the same study when DOB is not in effect. The phase portraits concerning both cases are displayed in Fig. 3.

As may be examined in Fig. 3(a), the system maintains its balance upon being perturbed. Regardless of the amplitude of the perturbation within the  $\pm 0.5$  Nms range, the rate change of angular momentum eventually converges and the robot recovers its balance. In our simulation studies, the proposed controller attenuated disturbances up to  $\pm 0.9$  Nms. On the contrary, the system could not be stabilized when the DOB is not in effect. Its rate change of angular momentum grows exponentially; see Fig. 3(b). This intermediate result indicates the stabilization capability of the proposed centroidal momentum observer method.

## III. SIMULATION EXPERIMENTS

In order to implement the proposed centroidal momentum observer, a series of simulations were performed using the 3D dynamic simulator MSC ADAMS. Simulations were executed using a PC that possesses an Intel (R) Xeon (R) W-2155 processor (3.3 GHz) with a 32.0 GB RAM, running on a 64-bit operating system. As a robot model, we used the one-legged robot model presented in [25]. The actual robot and its MSC ADAMS model is displayed in Fig.6, whereas the robot joint configuration was provided in Fig.1.

In our simulations, we considered 3 distinct scenarios in which different types of perturbations were applied:

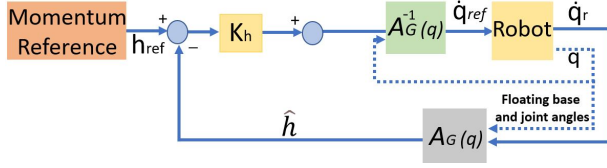


Fig. 4. Controller Model of Centroidal Momentum Control. The momentum reference is determined by trajectory planning.

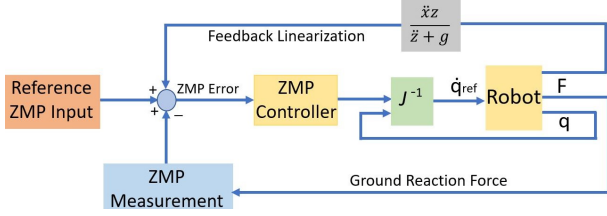


Fig. 5. Controller Model of ZMP Feedback Control with feedback linearization

Scenario 1: A 5 kg extra load was attached on the robot torso and the controllers were not aware of this load. In other words, this extra load was not considered in robot dynamics and CMM matrix computations. In this scenario, we test the controllers' ability to withstand parametric uncertainties.

Scenario 2: External forces along x-axis and z-axis were exerted on the robot. An external impulsive force of 30 N along the x direction and 100 N along the z direction were applied to the torso of the robot when  $t = 2$  s.

Scenario 3: The robot was subject to external momenta. When  $t = 2$  s, an external angular momentum in the form of step function with a 0.5 Ns amplitude was applied along the y-direction, and an external linear momentum in the form of step function with a 0.2 Ns amplitude was applied through the x-direction.

For each scenarios, three controllers were implemented and their respective performances were compared, namely, Centroidal Momentum Control [17] (CMC, see Fig.4), the proposed controller Centroidal Momentum Observer (CMD, see Fig.2) and ZMP Feedback Control (ZMP, see Fig. 5). The ZMP feedback controller includes a feedback linearization loop with a PI controller. The gain values concerning the controllers were tabulated in Table I. The results are displayed in Figs. 7-15, in which black dashed lines indicate reference variation, while solid blue, green, and red lines show response variations for the controllers CMD, CMC, and ZMP, respectively.

|     | Scenario 1 |       | Scenario 2 |     | Scenario 3 |  |
|-----|------------|-------|------------|-----|------------|--|
|     | Parameter  | Value |            |     |            |  |
| CMC | $K_h$      | 1     | 1          | 1   |            |  |
|     | $K_f$      | 0.3   | 0.3        | 0.3 |            |  |
| CMD | f          | 30    | 30         | 40  |            |  |
|     | $k_p$      | 0.6   | 0.6        | 0.6 |            |  |
| ZMP | $k_i$      | 4.3   | 4.3        | 4.3 |            |  |

TABLE I  
THE GAIN VALUES OF THE CONTROLLERS

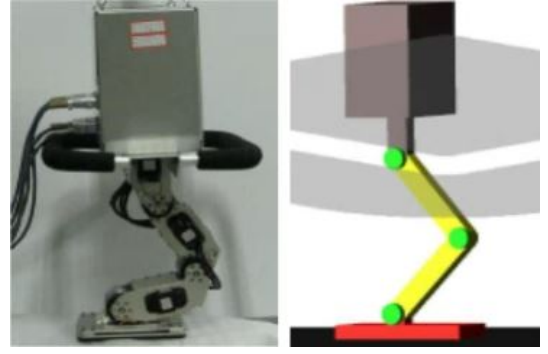


Fig. 6. The actual one-legged robot [25], and its simulation model.

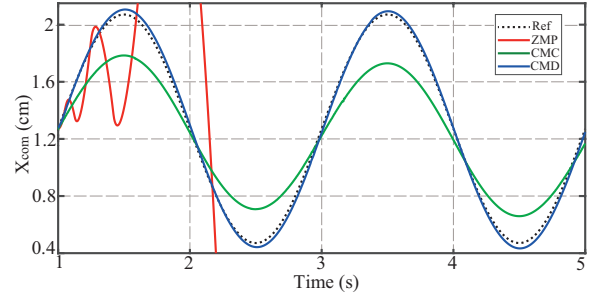


Fig. 7. The center of mass(CoM) on the x-axis ( $X_{com}$ ), Scenario #1.

#### A. Robot Model

The one-legged robot used in this study consists of 3 active joints, including a hip joint, a knee joint and an ankle joint, all along the pitch axis.  $q_i (i = 1, 2, 3)$  denote the relative angles between the links  $(i - 1)$  and the link  $(i)$ .  $\dot{q}_i (i = 1, 2, 3)$  represents joint velocities corresponding to each angle. Fig. 6 displays the actual robot and its simulation model.

#### B. Scenario #1 Result: Unknown Load

In Figs. 7, 8 and 9, the the x-axis CoM position ( $X_{com}$ ), the z-axis CoM position ( $Z_{com}$ ), and the x-axis ZMP position ( $X_{zmp}$ ) are displayed for this scenario, respectively. As may be observed, the ZMP controller could not stabilize the robot in this scenario, the robot tip over. In particular, the x-axis ZMP reached to the rear end of the support polygon. On the contrary, both CMD and CMC could stabilize the robot. Among them, CMD outperformed CMC. The RMS (Root Mean Squares) values of CoM tracking error are 0.0267 cm and 0.193 cm for x-axis and 0.0922 cm and 0.473 cm for z-axis, concerning CMD and CMC, respectively. Likewise, the RMS value of  $X_{zmp}$  variation in the case of CMD is 3.054 cm. It is measured as 4.765 cm, for the case of CMC. This results suggest that CMD is able to handle parametric uncertainties.

#### C. Scenario #2 Result: External Forces

In Figs. 10, 11 and 12, the the x-axis CoM position ( $X_{com}$ ), the z-axis CoM position ( $Z_{com}$ ), and the x-axis ZMP position ( $X_{zmp}$ ) are displayed for this scenario, respectively. In this case, all three controllers achieved dynamic balancing. Similar

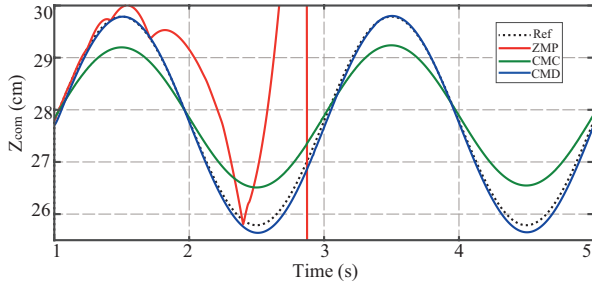


Fig. 8. The center of mass(CoM) on the z-axis ( $Z_{com}$ ), Scenario #1.

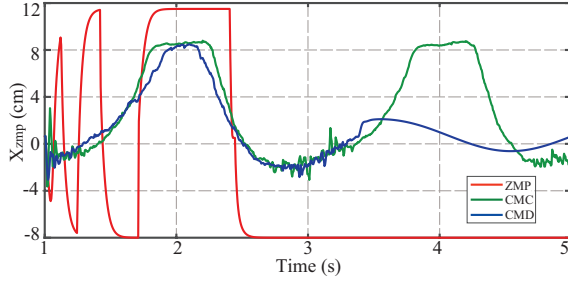


Fig. 9. ZMP position on the x-axis ( $X_{zmp}$ ), Scenario #1.

to Scenario #1, CMD outperformed CMC and ZMP. CoM tracking errors are 0.02736, 0.187, and 5.137 for x-axis and 0.0562, 0.468, and 0.703 for z-axis, concerning CMD, CMC, and ZMP, respectively. The RMS values of concerning the actual  $X_{zmp}$  are 7.324, 7.717, and 3.756 for CMD, CMC, and ZMP, respectively. Judging from these results, CMD appears to provide more favorable balancing performance when the robot is subject to external forces, despite the variation in  $X_{zmp}$ .

#### D. Scenario #3 Result: External Momentum

In Figs. 13, 14 and 15, the the x-axis CoM position ( $X_{com}$ ), the z-axis CoM position ( $Z_{com}$ ), and the x-axis ZMP position ( $X_{zmp}$ ) are displayed for this scenario, respectively. In this scenario, the controllers CMC and ZMP failed to stabilize the robot and is tip over, whereas CMD was able to maintain dynamic balance. In order to counteract the external angular momentum applied to the CoM, the robot automatically bent his torso in the opposite direction and maintain the balancing. The RMS values of  $X_{com}$  and  $Z_{com}$  error are 0.0232 and 0.0564 when implementing CMD. In addition, the RMS of  $X_{zmp}$  was 3.957 for this controller.

#### IV. CONCLUSION

In this paper, a controller based on centroidal momentum feedback with DOB is proposed to achieve continuous and dynamically balanced motion for a 4-link 3-DoFs single-legged robot. The performance of this controller was compared with a classical centroidal momentum feedback controller and a ZMP feedback controller which are used widely. The comparison study was realized considering 3 distinct scenarios in which different types of perturbations were applied to the robot. As the main result, the proposed CMD controller outperformed

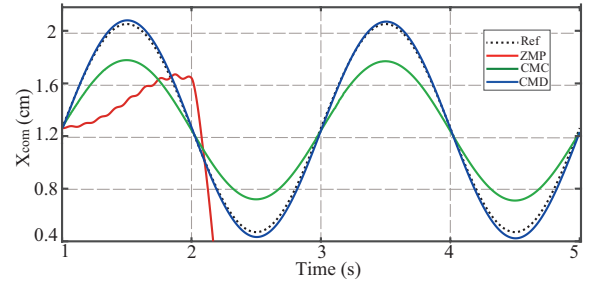


Fig. 10. The center of mass(CoM) on the x-axis ( $X_{com}$ ), Scenario #2. In this figure,  $X_{com}$  varied between 2.1 and  $-10$  cm and therefore truncated after  $t = 2$  for a clear illustration.

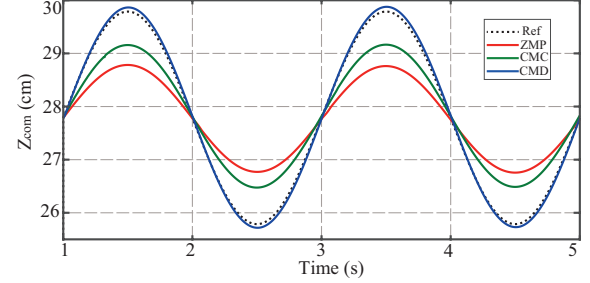


Fig. 11. The center of mass(CoM) on the z-axis ( $Z_{com}$ ), Scenario #2.

other controllers as it provided relatively more robust and favorable performance. In our next work, we will examine the experimental performance of CMD in our lower body exoskeleton system. We will also investigate practical issues of CMD application, e.g., sensor noise and state estimation regarding the absolute velocity of the floating base [29].

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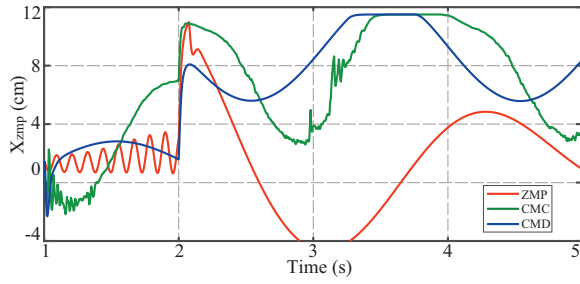


Fig. 12. ZMP position on the x-axis ( $X_{zmp}$ ), Scenario #2.

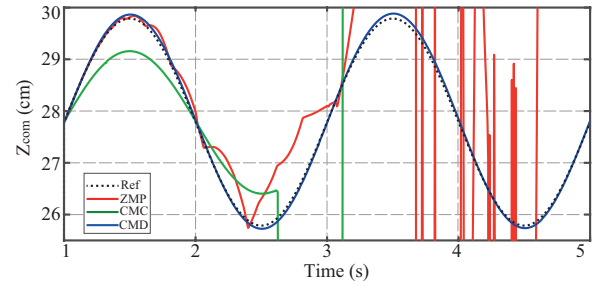


Fig. 14. The center of mass(CoM) on the z-axis ( $Z_{com}$ ), Scenario #3.

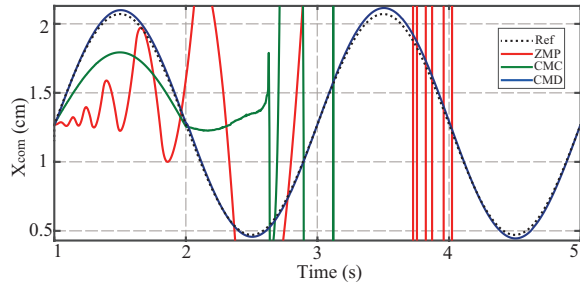


Fig. 13. The center of mass(CoM) on the x-axis ( $X_{com}$ ), Scenario #3.

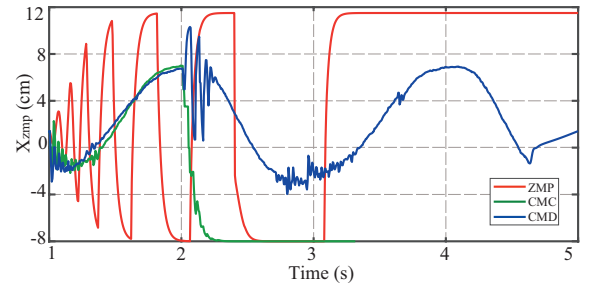


Fig. 15. ZMP position on the x-axis ( $X_{zmp}$ ), Scenario #3.

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